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STOCHASTIC FILTERING AND CONTROL. (U)  
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**STOCHASTIC FILTERING AND CONTROL**

**Progress Report on AFOSR Grant 77-3177**

**Principal investigator: T. E. Duncan**

**AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)**

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# ABSTRACT

During the past year we have studied problems in stochastic filtering and control. Some stochastic filtering problems for continuous processes in manifolds have been solved and necessary and sufficient conditions for an optimal control of a continuous stochastic system in a sphere bundle have been obtained. Jump processes have been formulated in a Riemannian tangent bundle and estimation and control problems have been solved for such stochastic systems. Brownian motion has been used to compute local formulae for some index theorems from the asymptotics of the heat equation. Some geometrical properties of linear systems have been investigated.

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## Research Activity

Supported by the Grant during the Past Year

During the past year we have investigated a number of topics in stochastic systems. Some stochastic filtering problems were modelled where the observations evolve in a compact Riemannian manifold and the state evolves in a Euclidean space and a stochastic equation was obtained for the estimate of the state [3]. A special case for this model was solved explicitly using the linear filtering results of Kalman-Bucy.

Some stochastic processes were defined in the tangent bundle of a Riemannian manifold that had the property that the fibre portion of the process was a jump process. This description required an invariant formulation of a jump process. With these processes problems of estimation [5] and control [6] were formulated and solved. Since the fibres of the tangent bundle are linear spaces some results for jump processes in linear spaces could be used.

Some continuous stochastic systems in the sphere bundle of a Riemannian manifold were defined by solutions of stochastic differential equations and necessary and sufficient conditions were given for an optimal control using dynamic programming (Hamilton-Jacobi) methods [4].



A probabilistic approach with Brownian motion has been used to investigate the heat equation techniques that have been used to obtain local formulae in a compact Riemannian manifold for various index theorems [7].

Recently Byrnes-Duncan [2] have investigated the topological properties of the family of real rational functions formed by ratios of  $(n-1)$  degree polynomials and  $n^{\text{th}}$  degree monic polynomials without common factors (e.g. minimal scalar input-output transfer functions). This investigation was initiated by Brockett [1]. Byrnes-Duncan reformulated this problem so that these rational functions were described by a subset of a linear space so that homotopy constructions were trivial. Our results include those of Brockett as well as some others.

During this academic year C.-P. Tsai is a half-time research associate who is supported by this grant. Previously he has studied the perturbation of the stochastic linear regulator problem [9] and has obtained with Fleming [8] some asymptotic estimates of the exit probability of a stochastic control system from a domain in  $\mathbb{R}^n$ . Currently we are working on problems in this latter area.

## REFERENCES

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3. T.E. Duncan, Some filtering results in Riemann manifolds, to appear in Information and Control 35 (1977)
4. T.E. Duncan, Optimal control of stochastic systems in a sphere bundle, to appear in Lecture Notes in Math., Springer-Verlag, New York
5. T.E. Duncan, Estimation for jump processes in the tangent bundle of a Riemann manifold, submitted for publication
6. T.E. Duncan, Optimal control of continuous and discontinuous processes in a Riemannian tangent bundle, submitted for publication
7. T.E. Duncan, The heat equation, the Kac formula and some index theorems, submitted for publication
8. W.H. Fleming and C.-P. Tsai, Optimal inclusion probability and differential games, preprint
9. C.-P. Tsai, Perturbed stochastic linear regulator problems, to appear in SIAM J. Control and Optimization



### Other Activities Related to the Grant

The principal investigator was an invited participant at the following conferences:

1. P.D.E. and Geometry Conference, Park City, Utah, February, 1977
2. Seventh Annual Communication Theory Workshop, Tucson, Arizona, April, 1977
3. Conference on Measure Theory-Applications to Stochastic Analyses in particular to Problems of Stochastic Filtering and Control, Oberwolfach, West Germany, July, 1977